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AN INVESTIGATION OF TWO SAFE

ESCAPE FROM BASE FLIGHT PROFILES.

ROBERT A. RAPPOLD LT COLONEL, USAF

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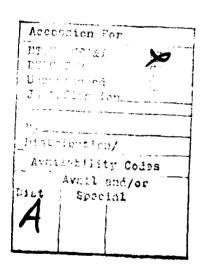
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This research establishes two base case scenarios for "safe escape" profiles			
for large conventional aircraft. The profiles considered were: (1) a constant			
altitude dash, and (2) a constant airspeed climb. The flight profile modeling			
assumed the aircraft had first reached a safe maneuvering airspeed and altitude.			
Other assumptions were consistent with aerodynamic and pilot limitations and operational considerations. The governing differential equations of motion			
are derived and the Runge-Kutta numerical solution technique applied.			
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## AN INVESTIGATION OF TWO SAFE ESCAPE FROM BASE FLIGHT PROFILES

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January 1981

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#### PREFACE

The research reported here was in support of research efforts for the Air Force Weapons Laboratory, Kirtland AFB, N.M. Of primary interest was the study and subsequent modeling of two rational flight profiles: a low-level dash, and a constant airspeed climb. These efforts were part of the larger problem concerned with "Safe Escape from Base" for aircraft located at fields alerted for nuclear attack. This research reports the results of the study and also contributes to the basic understanding of the larger safe escape problem.

#### SUMMARY

The purpose of this research was to establish two base case scenarios for "escape" profiles for large conventional aircraft. The two profiles considered were: (1) a constant altitude dash, and (2) a constant airspeed climb. The results of these two profiles are presented in the report

Several assumptions were made in the modeling of the two profiles.

The first assumes the aircraft has reached a safe maneuvering airspeed and altitude. From this point the models are employed and generate the results shown in the figures. The remainder of the assumptions are concerned with the type of aircraft considered, aerodynamic and pilot limitations, and operational considerations.

FORTRAN programs are available of the two models.

Thanks must go to Major Roy R. Kilgore, and Captain Joseph B. Williams, formerly of the Department of Mathematical Sciences, and ClC W. L. Troy for their efforts in developing the math model.

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#### SECTION 1

#### BACKGROUND AND INTRODUCTION

The necessity to plan for a submarine launched ballistic missile attack on the U.S. military forces has many far reaching implications. In particular, the deployment of aircraft becomes sensitive to the crucial time variable - warning time, missile flight time, aircraft response time, and takeoff and escape time. All of these variables and their interactions are being studied in the context of the "Safe Escape from Base" problem. An integral part of this larger problem is the different profiles aircraft may use after takeoff to "escape" from the parent base under attack.

The research reported here is the result of studying and modeling two of the possible "escape" profiles. One profile is concerned with obtaining the maximum distance in a horizontal sense; in that, once a safe maneuvering airspeed and altitude have been reached, the pilot maintains the altitude and begins a low-level dash away from the base. The other profile attempts to maximize the distance in a vertical direction by continuing the climb once a safe airspeed and altitude have been reached. Both of these profiles will be examined in a setting that is consistent with physical limitations and current operational procedures.

The reaction and escape time is more crucial to larger, slower aircraft; e.g., big bombers, tankers and cargo type. As such, the developed models will use these types of aircraft as guides in making judgements on certain characteristics and parameter values. The following six assumptions and explanations will be used to define the rational maneuver regime used in this research:

- Acceleration is nonnegative (normal acceleration) prevents loss of airspeed and high speed stalls.
- Max allowable "g" is the aircraft structural limit "g" prevents structural damage.
- 3. Max  $C_L \leq C_L$  for specific aircraft prevents stalls.
- 4. Max q (=  $\frac{\rho V^2}{2}$  ) will not be exceeded prevents structural damage.
- Aircraft will have thrust to weight ratio less than one simplifies the math model.
- 6. The models will start at a safe maneuvering airspeed and altitude.

Other specific assumptions will be made as needed in the development of the math models given in the appendices.

The remainder of this report is organized as follows: Section 2 describes and gives results of the constant altitude profile, Section 3 examines the constant airspeed profile, and Section 4 contains the conclusions. In addition, Appendices A and B give the development of the math models for the constant altitude and constant airspeed profiles, respectively.

#### SECTION 2

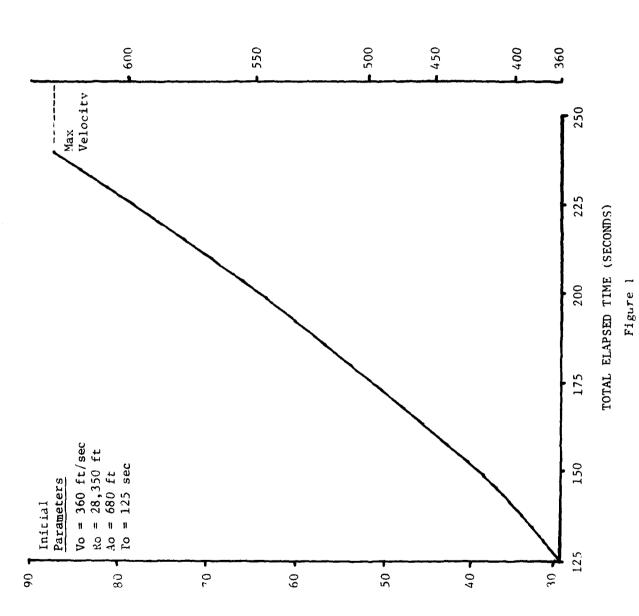
#### CONSTANT ALTITUDE PROFILE

One of the profiles that should be considered when evaluating various flight paths of escape from a base under attack is the low-level dash. In this profile the horizontal distance from the base is maximized. This is accomplished by climbing to a safe altitude, then leveling off and dashing as long as necessary. For the purposes of this research the assumptions are made that in 125 seconds the aircraft can climb to 680 feet, reach a velocity of 360 feet/second and reach a horizontal distance of 28,350 feet from the base. These initial parameter values define the safe maneuvering airspeed and altitude regime. Additionally, these values will be used as inputs to the constant altitude dash model.

Given the above safe maneuvering regime, the constant altitude model then numerically solves the governing differential equation. Appendix A develops this model and presents the numerical solution technique. Figure 1 presents the results of the constant altitude profile. The figure starts at 125 seconds on the time axis because this is the time the model is "turned-on" to compute velocity and distance. The left-hand axis gives the horizontal distance from the base as a function of time. The right-hand axis gives the velocity as a function of time and also the distance. Special mention should be made of the dashed line labeled max velocity. At this time (approximately 240 seconds) the maximum velocity of this particular aircraft has been reached (approximately 630 feet/second). If the distance from the base must be known for times greater than 240 seconds a linear equation will give the results. For example, suppose we want the distance, D, at time T, T greater than 240, then:

$$D = 630 \times (T-240) + 87500. \tag{1}$$





Distance From Base (Feet X  $10^3$  )

This completes the presentation of the constant altitude model.

Basically, any aircraft can be modeled with this approach as long as the assumptions are not violated. Some of the high performance aircraft will violate certain assumptions and model modification would be required. We will now turn to the examination of a second "escape" profile.

#### SECTION 3

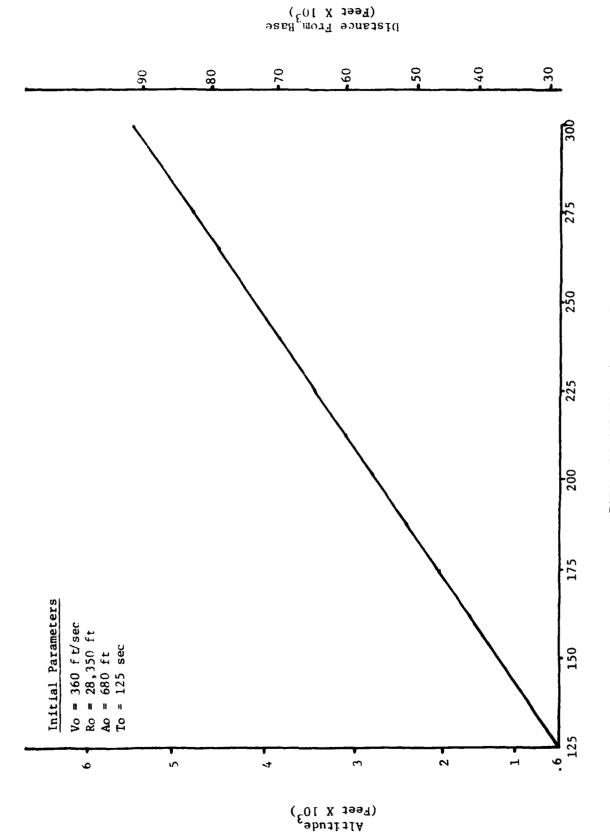
#### CONSTANT AIRSPEED PROFILE

The second rational "escape" profile to be considered is a constant airspeed climb. This path attempts to maximize the vertical distance from the base. This research will restrict the climb to one of constant airspeed.

Although this is not necessarily the optimum climb profile, it does represent current operational procedure and a rational profile considering pilot limitations.

Again the parameter values of 125 seconds, velocity of 360 feet/second, altitude of 680 feet, and distance of 28,350 feet will be assumed at the beginning of the climb. Instead of leveling-off and dashing, the profile now requires a continuation of the climb with a constant airspeed. Appendix B develops this model and presents the solution technique. Figure 2 presents the results of the constant airspeed profile. This figure also starts at 125 seconds because of the time required to reach the safe maneuvering regime. The left-hand axis gives the altitude as a function of time. The right-hand axis gives the horizontal distance from the base as a function of time and altitude. A reminder should be made that this research was accomplished on large, relatively slow aircraft. The figure stops at five minutes total elapsed time only for purposes of illustration. The model could continue to calculate the altitude and distance as long as no model assumptions were violated.

This completes the presentation of the constant airspeed model. Any aircraft could also be modeled with this approach as long as the assumptions are not violated.



TOTAL ELAPSED TIME (SECONDS)
Figure 2

#### SECTION 4

#### CONCLUSIONS

We have shown the results of two safe maneuvering "escape" profiles. Although several simplifying assumptions were made in Section 1 and the Appendices, the developed models have a wide range of application to larger, slower aircraft. The purpose of this research was to develop a base case scenario which will allow the Air Force Weapons Laboratory to use it for further sophistication and to aid in other decisions regarding further study efforts. The results depicted on Figures 1 and 2 provide the results of the base case scenarios.

To accommodate higher performance aircraft the two models would have to be modified. The extent of the modification would depend on the specific aircraft and the performance parameters to be modeled. Basically, any changes to the models could be limited to the subroutines of the Fortran programs.

#### APPENDIX A

#### DEVELOPMENT OF CONSTANT ALTITUDE MODEL

The inputs to this model assume the aircraft has reached a safe maneuvering airspeed and altitude. This assumption is characterized by the initial quantities:

To: elapsed time from take-off to safe maneuvering regime

Vo: velocity at time To

Ao: altitude at time To

Ro: horizontal distance from base at To.

We will now assume the aircraft levels-off and maintains the altitude, Ao, and accelerates away from base to maximize the horizontal distance. With this profile a simplified version of the equation of motion reduces to:

where: m = aircraft mass

r = aircraft longitudinal acceleration

T = total thrust

D = total drag.

For the large conventional aircraft considered in this research.

$$D = (C_{D_0} + KC_L^2) \frac{\rho v^2}{2} s,$$
 (A-2)

where:  $C_{D_O}$ : parasite drag coefficient

 $KC_{\tau}^{2}$ : induced drag coefficient

ρ: air density

V: velocity

S: wing area.

The quantities  $\mathbf{C}_{\mathbf{D}_{\mathbf{O}}}$ , K,  $\mathbf{C}_{\mathbf{L}}$ , and T may all depend on the compressibility or mach number. However, for every aircraft, within certain limits, the following relationships hold:

T is approximately constant

 $C_{\underline{I}}$  is proportional to  $\alpha$  (angle of attack)

 $\mathbf{C}_{\mathbf{D}_{\mathbf{O}}}$  is approximately constant

K is approximately constant.

During a level flight dash, lift = weight, thus,

$$\frac{\rho v^2 s}{2} c_L = W \tag{A-3}$$

or

$$C_{I.} = \frac{2W}{\rho V^2 s} \tag{A-4}$$

With V = r and substituting equations A-2 and A-3 into A-1 the equation of motion, after rearranging, becomes:

$$m \ddot{r} + C_{D_0} \frac{\rho s}{2} (\dot{r})^2 + \frac{2KW^2}{\rho s} \frac{1}{(\dot{r})^2} = T$$
 (A-5)

With the assumptions made earlier (Section 1) and the simplifying assumptions made in this appendix, equation A-5 is the differential equation governing the equation of motion for the low-level dash. It now remains to solve the equation.

It is possible to solve equation A-5 in closed form with some further simplifying assumptions. However, in an attempt to maintain a sense of generality the equation will be solved numerically by Runge-Kutta. With the additional assumption of a standard day, the only dependent variables in equation A-5 are m, W (due to fuel loss), r and  $\ddot{r}$ . Weight is related to mass; thus the dependent variables are m,  $\dot{r}$ , and  $\dot{r}$ . The Runge-Kutta technique works on first order differential equations: thus equation A-5 will be treated as a first order equation in  $v = \dot{r}$ . Upon rearranging, the equation to be solved may be expressed as: v = f(t, m, v) (A-6)

where t represents the independent time variable. If the time step,  $\Delta t$ , is sufficiently small the position can be calculated from the expression

$$R_{\text{new}} = R_{\text{old}} + (\Delta t) v$$
 (A-7)

rather than the differential equation  $\dot{R} = v$  which necessitates a second Runge-Kutta.

The problem to be solved can now be expressed as:

$$\dot{v} = f (t, m, v)$$

$$v (t) = V$$

and

$$R_{\text{new}} = R_{\text{old}} + (\Delta t)_{\text{V}}$$

with R initially set to Ro.

Since the time step will be chosen sufficiently small to enable the use of equation A-6 to determine the position, the mass and weight will be calculated at the beginning of each iteration rather than internal to the Runge-Kutta calculation. Furthermore, since t does not explicitly appear in equation A-6 the simplified notation below is more suggestive of the true problem.

$$\dot{\mathbf{v}} = \mathbf{f} \quad (\mathbf{v}). \tag{A-8}$$

The time variable t will now be indexed for  $i = 0, 1, 2, ..., t_{i+1} = t_i + \Delta t$  and the Runge-Kutta scheme for  $V_i$  is:

$$v_{i+1} = v_i + \frac{1}{6} (K_1 + 2K_2 + 2K_3 + K_4)$$
 (A-9)

where;

$$K_1 = (\Delta t) f(v_i)$$
 $K_2 = (\Delta t) f(v_i + .5 K_1)$ 
 $K_3 = (\Delta t) f(v_i + .5 K_2)$ 
 $K_4 = (\Delta t) f(v_i + K_3)$ 

Summarizing the solution methodology we have:

- 1. Known initial conditions To, Vo, and Ro
- 2. Index the time variable,  $t_{i+1} = t_i + \Delta t$ , i = 0, 1, 2, ...
- 3. Calculate  $\mathbf{m}_{\,\mathbf{i}}$  and  $\mathbf{W}_{\,\mathbf{i}}$  from the fuel loss
- 4. Calculate  $v_{i+1}$  from equation A-9
- 5. Calculate  $R_{i+1} = R_i + (\Delta t) v_i$ .

This scheme is repeated until the calculated velocity reaches the maximum velocity of the specific aircraft under study.

#### APPENDIX B

#### DEVELOPMENT OF CONSTANT AIRSPEED MODEL

The inputs to this model assume the aircraft has reached a safe maneuvering airspeed and altitude. This assumption is characterized by the initial quantities:

To: elapsed time from take-off to safe maneuvering regime

Vo: velocity at time To

Ao: altitude at time To

Ro: horizontal distance from base at To.

We will now assume the aircraft maintains the true airspeed Vo and continues to climb at an angle  $\gamma$ . In what follows the small variations with altitude and velocity will be neglected. With a climb angle  $\gamma$  a simplified version of the governing force yields:

$$L = W \cos \gamma \tag{B-1}$$

where: L = lift

W = weight

and 
$$T - D = W \sin \gamma$$
 (B-2)

where: T = thrust

D = drag.

For the large conventional aircraft considered in this research, the climb angle is small; thus upon rearranging equation B-2 we have:

$$\sin \gamma \approx \gamma = \frac{T-D}{W}$$
 (B-3)

To continue with the assumptions for the aircraft considered here, we have

$$D = (C_{D_0} + KC_L^2) \frac{\rho V^2}{2} s$$
 (B-4)

where:  $C_{\overline{D}}$ : parasite drag coefficient

 $KC_1^2$ : induced drag coefficient

ρ: air density

V: velocity

S: wing area

The quantities  $\mathbf{C}_{\mathbf{D}_0}$ , K,  $\mathbf{C}_{\mathbf{L}}$ , and T may all depend on changes in altitude and speed. However, within the limits of this research, we will assume the following relationships hold:

T is approximately constant

 ${\rm C}_{L}^{}$  is proportional to  $\alpha$  (angle of attack)

 $^{\mathrm{C}}\mathrm{D}_{\mathrm{O}}$  is approximately constant

K is approximately constant

Since the climb angle  $\gamma$  is small (cos  $\alpha \approx 1$ ) we have from equation B-1 that lift is approximately equal to weight. Thus,

$$\frac{\rho v^2 s}{2} C_L = W \tag{B-5}$$

or 
$$C_L = \frac{2W}{\rho v^2 s}$$
 (B-6)

Substituting equations B-4 and B-6 into B-2, we arrive at the expression for the climb angle:

$$T = \frac{1}{W} \left[ T - \frac{1}{2} c_{D_0}^2 \rho V^2 S - \frac{2KW^2}{\rho V^2 S} \right]$$
 (B-7)

Now let  $\boldsymbol{v}_{\!\!\!\boldsymbol{v}}$  be the vertical component of velocity and  $\boldsymbol{v}_{\!\!\!\boldsymbol{h}}$  the horizontal component. Then,

$$V_{v} = V \sin \gamma \approx V \gamma$$
 (B-8)

and

$$V_h = V \cos v \approx V.$$
 (B-9)

If the time step is chosen sufficiently small, instead of integrating to obtain altitude and distance we may use the expressions

$$A_{\text{new}} = A_{\text{old}} + (\Delta t) V_{\text{v}}$$
 (B-10)

and

$$R_{\text{new}} = R_{\text{old}} + (\Delta t) V_{\text{h}}. \tag{B-11}$$

We now have all the quantities to describe the solution methodology. First we have the known initial conditions To, Vo, Ao, and Ro. The time domain will now be divided into subintervals of length  $\Delta t$  and the system will be examined at the times  $t_{i+1} = t_i + \Delta t$ ,  $i = 0, 1, 2, \ldots$  For each i,  $\gamma_i$  will be evaluated since the weight is changing. Then the expressions for the vertical and horizontal components will be calculated and then finally the altitude and position will be determined. In algorithmic form, the technique is:

- 1. Known initial conditions To, Vo, Ao, and Ro.
- 2. Index the time variable,  $t_{i+1} = t_i + \Delta t$ , i = 0, 1, 2, ...
- 3. Calculate  $\gamma_i$  from equation B-7
- 4. Calculate  $V_{i}$  and  $V_{h}$  from equations B-8 and B-9 respectively
- 5. Calculate  $A_{i+1} = A_i + (\Delta t) V_{v_i}$
- 6. Calculate  $R_{i+1} = R_i + (\Delta t) V_{h_i}$

This scheme may be repeated almost indefinitely. However, before the maximum altitude is reached the simplifying assumptions will be violated. Thus, it is recommended that a maximum altitude be included in the scheme. Once this altitude is reached the model will have to be changed or the aircraft would have to level-off.

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